Common Course Outline
MATH 2700, FOUNDATIONS OF MATHEMATICS AND LOGIC: WRITING INTENSIVE

A. Course Description

1. Number of Credits: 4
2. Lecture hours per week: 4
   Lab hours per week: None
3. Prerequisite: MATH1520
4. Suggested: MATH2510, 2520 or 2400
5. MnTC Goals: Goal 4/ Mathematical/Logical Reasoning

Catalogue Description:
This course will be useful to all students pursuing advanced mathematics at four-year institutions, including but not limited to those intending majors in mathematics or mathematics education. Topics include basic logic, techniques of mathematical proof, set theory, relations and functions, sequences and series, and basic number theory. The course may include additional topics at the discretion of the instructor.

Writing is an important part of this course and will be comprehensively integrated into the course and will be a significant part of the course work and course grade. Writing proofs will be explained and practiced in the course and some assignments will be refined through revisions.

Recommended entry skills/knowledge: Mathematical maturity including an appreciation and recognition of the need for logically rigorous argument in mathematics.

B. Date last revised: April 2014

C. Outline of Major Content Areas

A. Basic propositional logic and first order predicate logic.
B. Basic set theory.
C. Techniques of mathematical proof.
D. Relations and Functions.
E. Cardinality of sets.
F. How to write Mathematical proofs in English.
G. Additional topics at the discretion of the instructor.

D. Course Learning Outcomes

Upon successful completion of MATH 2700, students will be able to:

A. Correctly use and analyze “and” “or” in logical statements. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
B. Recognize and analyze the use of logical quantifiers (e.g. existential and universal). (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
C. Construct and interpret truth tables for logical propositions. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
D. Manipulate union and intersection operators, including applications of DeMorgan’s Laws. (4a, 4b, 4c, 4d)
E. Construct Cartesian products of sets. (4a, 4b, 4c, 4d)
F. Recognize and construct proofs using direct methods. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)

G. Recognize and construct proofs using contrapositive methods. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
H. Recognize and construct proofs using contradiction methods. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
I. Recognize and construct proofs using mathematical induction methods. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
J. Recognize and construct proofs using algorithms (e.g. Division Algorithm, Euclidean Algorithm). (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
K. Disprove false statements by the method of counterexample. (2a, 2b, 2c, 2d, 4a, 4b, 4c, 4d)
L. Recognize an equivalence relation and construct equivalence classes. (4a, 4b, 4c, 4d)
M. Recognize and apply properties of relations, including equivalence relations, partial orderings, and total orderings. (4a, 4b, 4c, 4d)
N. Recognize when a relation is a function, and understand the properties of injectivity and surjectivity. (4a, 4b, 4c, 4d)
O. Apply operations of functions, including compositions and inverses. (4a, 4b, 4c, 4d)
P. Prove that sets have the same cardinality by exhibiting a one-to-one correspondence (i.e. bijection). (4a, 4b, 4c, 4d)
Q. Distinguish between countable (denumerable) and uncountable sets. (4a, 4b, 4c, 4d)
R. Understand and apply the Cantor diagonal argument to prove various sets are countable. (4a, 4b, 4c, 4d)
S. Use logical analysis to determine the structure of an assertion to be proved. (2a, 2b, 2c, 2d)
T. Use the structure of the assertion to structure the argument. (2a, 2b, 2c, 2d)
U. Determine validity and soundness of arguments. (2a, 2b, 2c, 2d)
V. Recognize and avoid fallacies. (2a, 2b, 2c, 2d)
W. Express arguments as English essays with correct sentence structure, subdivision into paragraphs, etc. (2a, 2b, 2c, 2d)
X. End proofs in conformity with their beginnings. (2a, 2b, 2c, 2d)
Y. Use correct grammar, punctuation, spelling and sentence structure for writing in English. (2a, 2b, 2c, 2d)
Z. Use accepted English mathematical terminology and idioms. (2a, 2b, 2c, 2d)
AA. Write with the mathematical voice including acceptable mathematical syntax, diction, punctuation, etc. (2a, 2b, 2c, 2d)
AB. Effectively use definitions. (2a, 2b, 2c, 2d)

E. Methods for Assessing Student Learning

Writing, including refinement through drafts, is an important part of this course and will be comprehensively integrated into the course and will be a significant part of the course work and course grade. The instructor will choose from among various evaluation techniques including – but not limited to – in-class testing, take-home testing, assignments, quizzes, attendance, group or individual projects, and research. The instructor will also choose a method for end-of-the-semester evaluation.

F. Special Information

This course is required for the NCC/Mankato Secondary Mathematics Education Program.
This course is required for the University of Minnesota, College of Science and Engineering, Math Major.